

WAVELET FORECASTING OF HUMIDITY AND TEMPERATURE

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ABSTRACT

Humidity and temperature are the main climate parameters that affect the earth environment and surface lives. Wavelet transforms provide spectral analysis of time series and extract abstract local information from the signal (data). In stationary wavelet transform (SWT) translation-invariance is achieved by up-sampling the filter coefficients so that faster algorithm to analyze a signal with more accuracy is obtained. The MATLAB wavelet toolbox performs a minimal right periodic extension leading to an extended signal of length 2^{j_0} , where j_0 is the maximum level of wavelet decomposition up to which the signal can be extended. The inverse SWT is used to obtain the extended signal from the predictions of components.

Key words: Humidity, Temperature, Approximation, Detail, Stationary Wavelet transform.

INTRODUCTION

Humidity represents the amount of water vapour present in air and plays an important role for the survival of surface life. For animal life depending on perspiration (sweating) to regulate internal body temperature, high humidity impairs heat exchange efficiency by reducing the rate of moisture evaporation from skin surfaces. This effect is measured in terms of heat index called humidex (Gaffen et al., 1999). Air temperature decides the amount of water vapour, the air can hold. Temperature and humidity affect people's comfort levels as well as their health. High humidity and heat means more water in the air, which can carry odor molecules further, leading to considerable stench in summer around bacteria sources such as garbage. Exercise regimens need to take into account temperature and humidity to avoid health risks. This is because the human body relies on evaporation of sweat to lead to cooling. If the air is both hot and humid, the body cannot evaporate the sweat as effectively, which can lead to dehydration, overheating

and even death. Researchers found that a joint effect exists between temperature and humidity on cardiovascular disease mortality. In conditions of low temperatures and high humidity, cardiovascular death rates increased. This could be due to high humidity affecting thrombotic risk, combined with the human body's various cold-stress responses (Eccel, 2012).

The Wavelet transform (WT) provides a useful decomposition of time series, in terms of both time and frequency, permitting us to effectively diagnose the main frequency component and to extract abstract local information from the time series (Antoine, 2004). WT has been frequently used for time series analysis and forecasting in the recent years. Models that accurately catch the statistical characteristics of the signal play a significant role in studying the network, in understanding its dynamics, in designing and controlling the network. We applied stationary wavelet transforms by up-sampling the filter coefficients for the extension of signal and best forecasts. A function $f(t)$ is decomposed into a set of basis (generating) functions $\psi_{j,k}(t)$ called wavelets as following:-

$$f(t) = \sum_{j \in \mathbb{Z}} \sum_{k \in \mathbb{Z}} c_j^k \psi_{j,k}(t) \quad (1.1)$$

The discrete wavelet coefficient,

$$\begin{aligned} c_j^k &= \langle f, \psi_{j,k} \rangle \\ &= \int_{\mathbb{R}} f(t) \psi_{j,k}^*(t) dt, \end{aligned}$$

where $\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k)$.

The sufficient condition for the reconstruction of any signal f of finite energy by the formula:-

$$f(t) = \sum_{j \in \mathbb{Z}} \sum_{k \in \mathbb{Z}} \langle f, \psi_{j,k} \rangle \psi_{j,k}(t)$$

is that the functions $\{\psi_{j,k}: j, k \in \mathbb{Z}\}$ form an orthonormal basis of $L^2(\mathbb{R})$. where j and k are integers representing the set of discrete translations and discrete dilations. We can write discrete wavelet transforms as:-

$$W_{j,k} f = \int f(t) 2^{j/2} \psi(2^j t - k) dt \quad (1.2)$$

These wavelets for all integers j and k produce orthonormal basis. We call $\psi_{j,k}(t) = \psi(t)$ as mother wavelet. Other wavelets are produced by translation and dilation of the mother wavelet. A multiresolution analysis consists of a sequence $V_j, j \in \mathbb{Z}$ of closed subspaces of $L^2(\mathbb{R})$ (Mallat, 1998; Kumar, 2017). We can express a function $f(x)$ in V_{j+1} spaces as following:-

$$f(x) = \sum a_{j+1}^k \phi_{j+1,k}(x)$$

Since $V_{j+1} = V_j \oplus W_j$, where,

$$V_{j+1} = \text{span} \left(\overline{\phi_{j+1,k}(x)} \right),$$

$$V_j = \text{span} \left(\overline{\phi_{j,k}(x)} \right)$$

$$W_j = \text{span} \left(\overline{\psi_{j,k}(x)} \right).$$

STATIONARY WAVELET TRANSFORMS

According to wavelet theory, any signal f can be decomposed into a set of scaling functions $\phi_{j,k}$ and wavelet functions $\psi_{j,k}; j, k \in \mathbb{Z}$, as following:-

$$f(t) = \sum_k a_j^k \phi_{j,k}(t) + \sum_{j_0}^j \sum_k d_j^k \psi_{j,k}(t) \quad (1.3)$$

where scaling coefficients,

$$\begin{aligned} a_j^k &= \langle f, \phi_{j,k} \rangle \\ &= \int f(x) \phi_{j,k}(t) dx, \quad \forall k \in \mathbb{Z} \end{aligned}$$

and wavelet coefficients,

$$\begin{aligned} d_j^k &= \langle f, \psi_{j,k} \rangle \\ &= \int f(t) \psi_{j,k}(t) dt \end{aligned}$$

are collectively known as approximation and detailed coefficients (Kumar et al., 2018; Kumar et al., 2019).

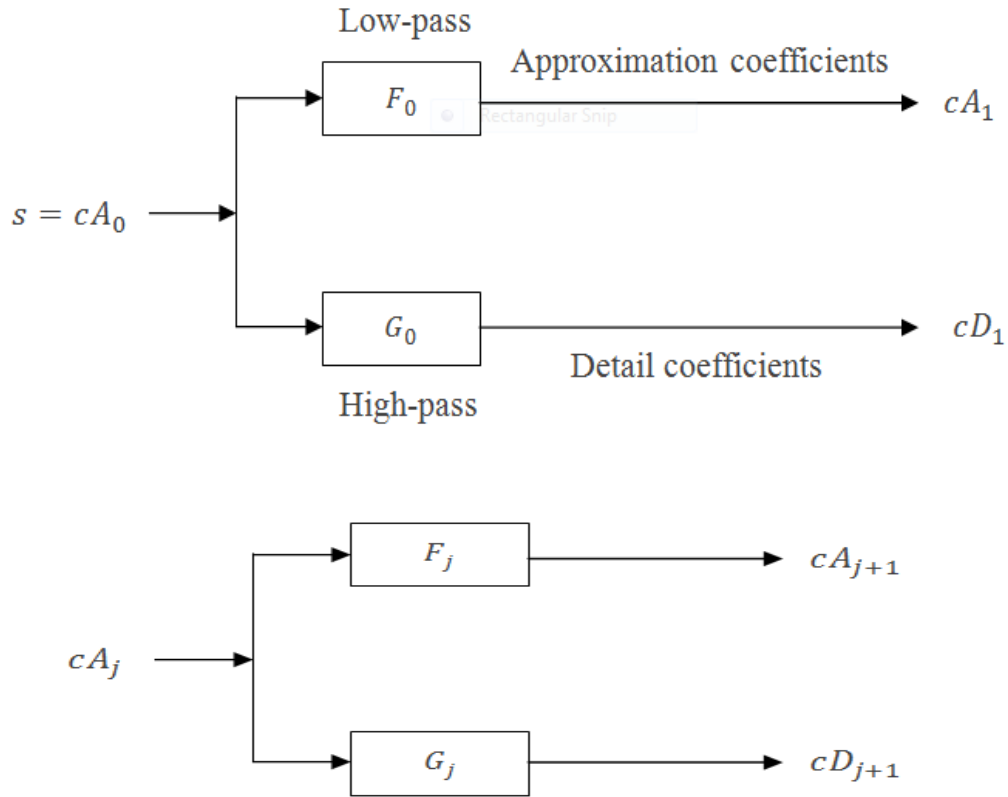


Figure 1: Decomposition of signal

Thus a given signal takes place a new version such as,

$$s = a_1 + d_1$$

Here a_1 is approximation and d_1 is detail of signal at various scale or time frames. Therefore, a signal s can be expressed as:-

$$s = \sum_k a_1^k \phi_{1,k}(t) + \sum_k d_1^k \psi_{1,k}(t) \tag{1.4}$$

The Stationary wavelet transform (SWT) is a wavelet transform algorithm designed to overcome the lack of translation-invariance of the discrete wavelet transform (DWT). Translation-invariance is achieved by removing the downsamplers and upsamplers in the DWT and upsampling the filter coefficients by a factor of 2^{j-1} in the j th level of the algorithm. The SWT is an inherently redundant scheme as the output of each level of SWT contains the same number of samples as the input, so that for a decomposition of N levels there is a redundancy of N in the wavelet coefficients. The SWT reconstructions result in lower error values and faster convergence compared to DWT. This is achieved by stationary wavelet transform (SWT) thresholding, which provides a translation-invariant basis (Nason et al., 1995). For SWT, a redundant decomposition can be obtained as:-

$$\tilde{\alpha}_{2^j}^{2^j k+p} = \langle f(t), 2^{-j/2} \phi(2^{-j}(t-p) - k) \rangle$$

$$\tilde{d}_{2^j}^{2^j k+p} = \langle f(t), 2^{-j/2} \psi(2^{-j}(t-p) - k) \rangle$$

where $p \in \{0, \dots, 2^j - 1\}$ allows for all the possible shifts in a discrete setting. For decomposition to j_m levels, 2^{j_m} different orthogonal bases can be generated. Each node in binary tree is indexed by parameters (j, p) , to which the set of coefficients $\left\{ \tilde{\alpha}_{2^j}^{2^j k+p} \right\}_{k \in \mathbb{Z}}$ is associated.

Each path from the root of the tree to a leaf corresponds to the set of functions,

$$\{2^{-j/2} \psi(2^{-j}(t-p_j) - k), k \in \mathbb{Z}, 1 \leq j \leq j_m\} \cup \{2^{-j_m/2} \psi(2^{-j_m}(t-p_{j_m}) - k), k \in \mathbb{Z}\}$$

which forms an orthogonal wavelet basis, resulting in a standard wavelet reconstruction. The inverse SWT is defined as the average of all the 2^{j_m} different reconstructions obtained in this manner. Unlike other extensions such as dual-tree wavelets, curvelets, and contourlets, these transforms are directly based on the standard wavelet transform. They are all based on the same wavelet and scaling functions and only differs in terms of shift and decimation. Our intention is to call attention to the advantages of the redundant shift-invariant version of the standard discrete wavelet transform, i.e., SWT, in comparison with its decimated versions, i.e., discrete wavelet

transforms (DWT) and discrete wavelet transforms with random shift (DWTRS), which is the most widely studied sparsifying transform and most commonly used in practice (Ye et al., 2004). The general step j convolves the approximation coefficients at level $j-1$, with upsampled versions of the appropriate original filters, to produce the approximation and detail coefficients at level j . This can be visualized in the following figure:-

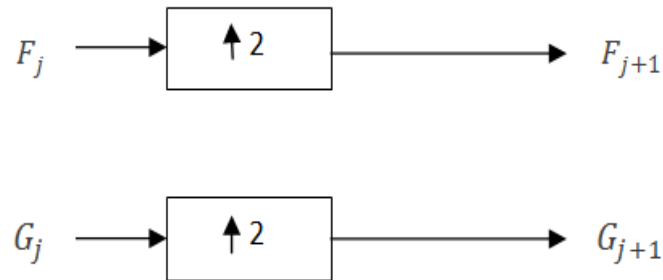


Figure 2: Filter computation by up-sampling

Models that accurately catch the statistical characteristics of the signal play a significant role in studying and understanding of climate dynamics.

STUDY AREA AND METHODOLOGY

Moradabad is a metropolitan area of Uttar Pradesh state in Northern India and is situated at the banks of Ramganga River. The latitudinal extent of city is $28^{\circ}20'N$ to $29^{\circ}15' N$ and longitudinal extent is $78^{\circ}4' E$ to $79^{\circ}E$. We have selected Moradabad region as our study area and analyzed its humidity and temperature as climate parameters from time period 01/10/2010 to 31/12/2018. The quantitative behaviour of humidity of Moradabad for given time period is shown in figure 3.

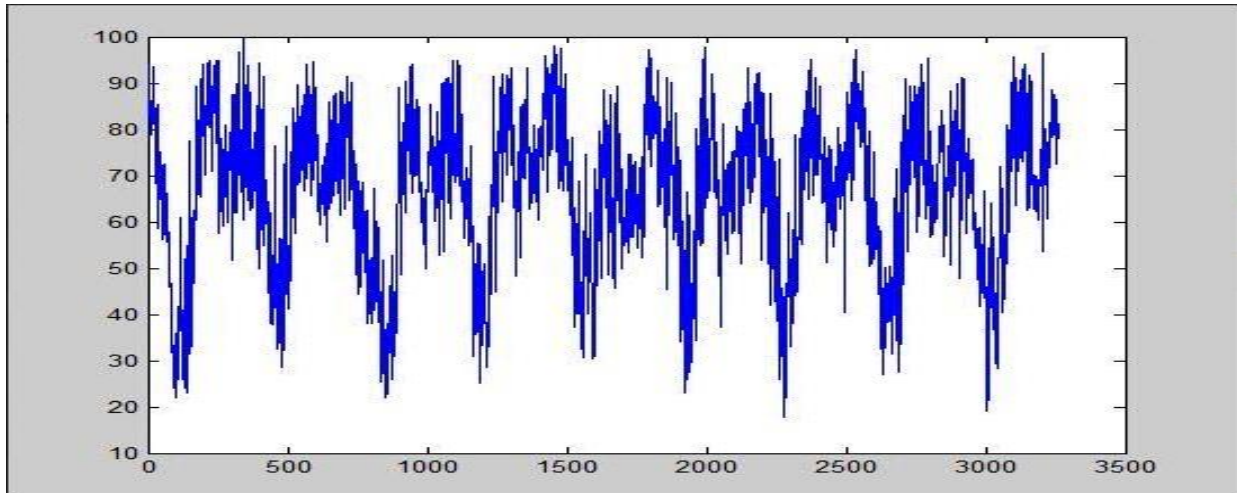


Figure 3: Humidity from 01/01/2010 to 31/12/2018

When the length of any signal is not divisible by 2^{j_0} , where j_0 is the maximum level of wavelet decomposition, the signal can be extended. The length of the above signal is 3260 and the decomposition level needed for SWT is 10, the tool performs a minimal right periodic extension. The tool performs a minimal right periodic extension leading to an extended signal of length 4096 (because 4096 is the smallest integer greater than 3260 and written in form of 2^{j_0}).

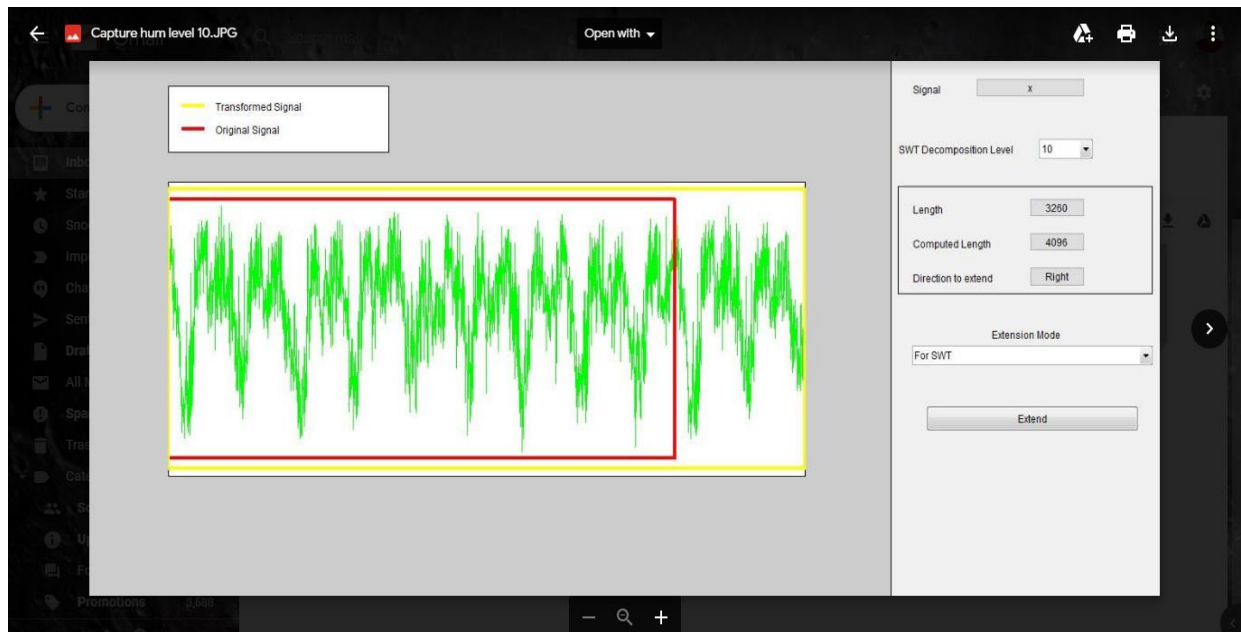


Figure 4: Extension of humidity using SWT

With help of discrete wavelet transforms the signal is decomposed in terms of approximation and wavelet coefficients. The stationary wavelet transform (SWT) is a wavelet transform algorithm designed to overcome the lack of translation-invariance of the discrete wavelet transform (DWT). Therefore, by removing the downsamplers and upsamplers in the discrete wavelet transforms and upsampling the filter coefficients by a factor of 2^{j-1} in the j th level of the algorithm, the translation-invariance is achieved. So, our approach is to decompose the original time series into scale or frequency related components and model each component separately, in order to obtain more accurate models. After obtaining the wavelet decomposition, we select the information from each level of decomposition for building the model. In the first phase we design predictive models for each of the decomposed components of the original series. In the second phase the developed forecasting models are used to predict future values for each component (Wong et al., 2003; Xiaohong et al., 2012). The inverse SWT is used in the second phase in order to obtain the forecasted signal from the predictions of the components. Same procedure is applied for the temperature of Moradabad as climate parameter for the same time period in figure 05 and 06.

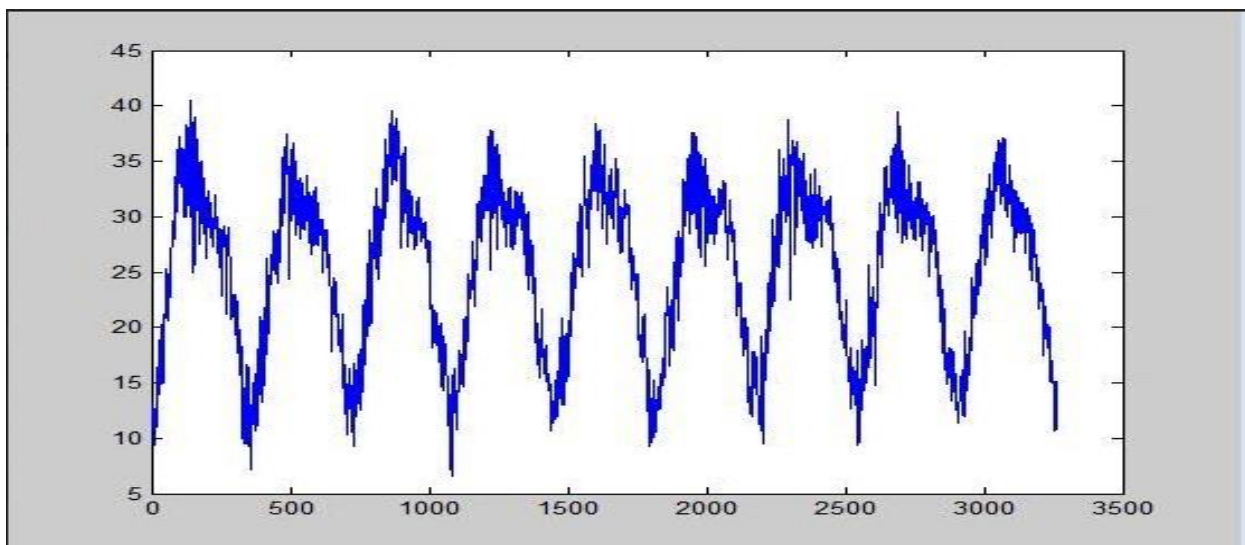


Figure 5: Temperature from 01/01/2010 to 31/12/2018

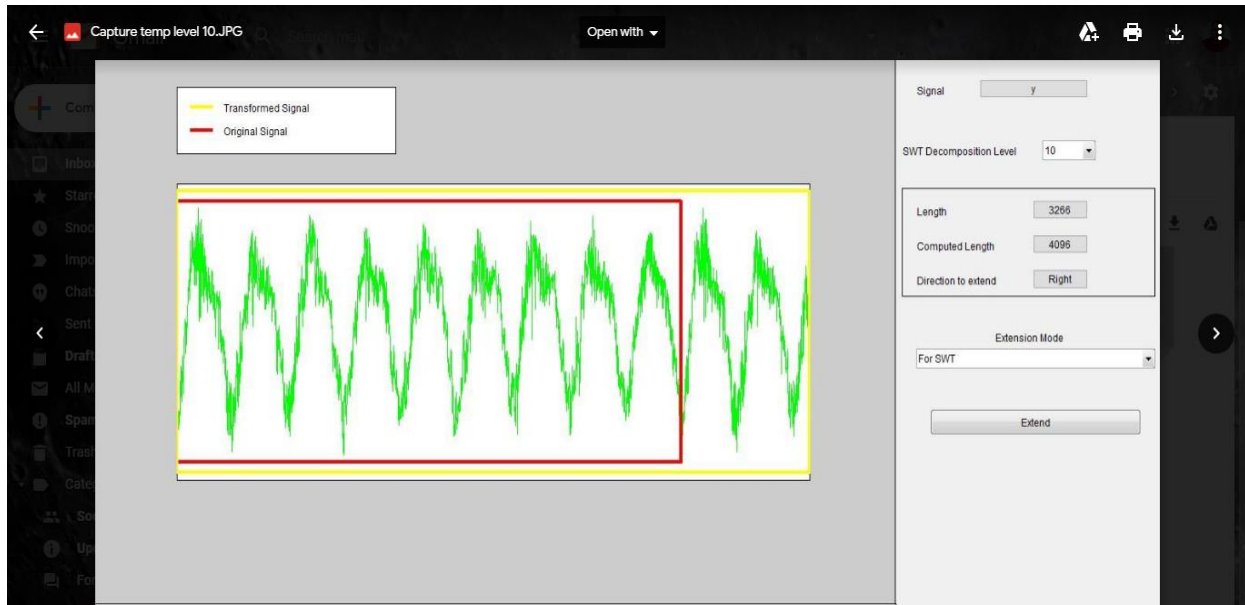


Figure 6: Extension of temperature using SWT

RESULTS AND DISCUSSION

In present work, we have analyzed average daily humidity and temperature behaviour of Moradabad region during the period 01/01/2010 to 31/12/2018 (9 years) by estimating different statistical parameters focusing climate change. From the trend of the signal, it is obvious that the humidity and temperature change periodically. With help of stationary wavelet transforms, the data of 9 years based upon average daily record is decomposed up to level 10 and extended up to 4096 points. Some statistical parameters of original and extended signal are as following:-

S.No.	Parameters	Original Signal		Extended signal	
		Humidity	Temperature	Humidity	Temperature
1	Average	66.4614	25.324	66.32514893	25.16496582
2	Skewness	-0.560177278	-0.412213786	-0.543498895	-0.39936
3	Kurt	-0.275144	-0.9660855	-0.332531978	-0.967927107
4	Standard Deviation	15.83924759	7.308807478	16.02691902	7.346746436
5	Correlation	-0.522998579		-0.408348038	

Table 1: Statistical parameters of original and extended signal

Skewness is a measure that studies the degree and direction of departure from symmetry. Negative value of skewness indicates that the humidity and temperature data is skewed to left. Skewed left means that left tail is long relative to the right tail. Kurtosis parameter measures the peakedness (or flatness) of the probability distribution of any signal (Rockinger et al., 2002). Low negative value of kurtosis indicates the weak intermittency in the humidity and temperature variability. Standard deviation indicates that how the data points are spread out over a wide range of values. Correlation describes the degree of linear relationship between two functions (or signals). The negative values of correlation means they are linearly related with negative slope and moderate value means that they are moderately dependent. The average and kurtosis of the extended signal are slightly less, while skewness and standard deviation are slightly greater than that of original signal for humidity and temperature both.

CONCLUSION

From the present analyses, we found that the humidity and temperature time-series of Moradabad in last 9 years is weakly intermittent. Skewness & Kurtosis parameters are low and negative, standard deviation is high and correlation is moderate & negative in that time period. With help of extended signal, we can say that in coming two years, the average value will be decreased, there will be little flatness or broadness in probability distribution, slight increment in degree and direction of departure from symmetry and data points will be slightly spread far from the mean value for humidity and temperature both. By virtue of these results, we can say that spectral analysis of humidity and temperature using stationary wavelet transforms provides a simple and accurate framework to investigate and forecast the climate behaviour.

ACKNOWLEDGEMENT

The financial support by Council of Scientific and Industrial Research (CSIR) for this work is greatly acknowledged.

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