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VIBRATION OF SQUARE PLATE WITH NON-LINEAR THICKNESS VARIATION

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ABSTRACT

Vibration of plates with variable thickness commonly used in modern technology to make the parts of air-crafts, fuselages, rockets, ship hulls etc. The aforementioned properties of composite tapered plates help scientists and researchers to reduce the vibration of system without compromising its integrity. Plates with thickness variability are of great importance in a wide variety of engineering applications.

Here, effect of Non-linear thickness variation is investigated on the vibration of non-homogeneous square plate with bi-parabolic temperature variation.. Square plate is assumed to be clamped at the boundary. Non-linear thickness variation is characterized linearly in one direction. Non-homogeneity is assumed as a linear variation of density of the plate's material. Rayleigh–Ritz method is used to derive frequency equation. Frequency is determined for first two mode of vibration.

Keywords: *Non-homogeneity, Square Plate, Thermal Gradient, Taper Constant.*

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Introduction

Vibration of plates with variable thickness commonly used in modern technology to make the parts of air-crafts,missiles, rockets etc. By carefully designing the thickness distribution, a substantial increase in stiffness and vibration capacities of the plate may be obtained over its counterpart. In aeronautical engineering, study of natural frequencies of non-homogeneous plates with thermal effect and variable thickness has been of great interest due to their utility in making the structural components of aircrafts designs etc. Many analysis show that plate vibrations are based on non-homogeneity of materials. Non-homogeneity can be natural or artificial. Many researchers [1-13] have analyzed the free vibration of visco-elastic plates with variable thickness.

In this paper , frequency is calculated for the first two mode of vibration of non-homogeneous tapered clamped square plate for different values of thermal gradient, taper constant and non-homogeneity constant with the help of latest software ' Mathematica'. Numeric results are tabulated for various combinations of parameters.

EQUATION OF MOTION AND ANALYSIS

Differential equation of motion for visco-elastic square plate of variable thickness in Cartesian coordinate is given by equation (2) [1]**:**

$[D_1(W,_{xxxx}+2W,_{xxyy} + W,_{yyyy}) + 2D_{1,x} (W,_{xxx}+W,_{xyy})+2D_{1,y} (W,_{yyy}+W,_{yxx})+$

$D_1,_{xx} (W,_{xx} +vW,_{yy})+D_{1,yy} (W,_{yy} + vW,_{xx})+2(1-v)D_1,_{xy} W,_{xy}]-vhp^2W=0$

(2)

Which is a different equation of transverse motion for Visco-elastic Square plate of variable thickness

Here, D_1 is the flexural rigidity of plate i.e.

$D_1 = Eh^3 /12(1-v^2)$ (2)

And corresponding two-term deflection function is taken as [4]

$W= [(x/a)(y/a)(1-x/a) (1-y/a)]^2 [A_1+A_2(x/a) (y/a)(1-x/a) (1-y/a)]$ (3)

Assuming that the square plate of engineering material has a steady two dimensional temperature distribution i.e.

□ □ □ $(1-x/a) (1-y^2/a^2)$ (4)

Where, denotes the temperature excess above the reference temperature at any point on the plate and denotes the temperature at any point on the boundary of plate and "a" is the length of a side of square plate.

The temperature dependence of the modulus of elasticity for most of the engineering materials can be expressed in this

$E = E_0$ (1- □ □ □ □ □ □ □

Where, E_0 is the value of the Young's modulus at reference temperature i.e. □ =0 and □ □ is the slope of the variation of E with □ □The modulus variation (5) become.

$E = E_0 [1-□ (1-x/a) (1-y^2/a^2)]$ (6)

Where, □ = □ □$_0$ □ □ □ □ □ □ □ thermal gradient.

It is assumed that thickness also varies linearly in two directions as shown below:

$h= h_0 (1+□_1x/a) (1+□_2y/a)$ (7)

Where, □□□ is taper parameter in x direction respectively and $h= h_0$ at x= y=0

Put the value of E & h from equation (6) & (7) in the equation (2) , one obtain

$D_1= [E_0 [1-□ (1-x /a) (1-y^2/a^2) h_0{}^3 (1+□_1x / a)^3 (1+□_2y/a)^3] / 12(1-v^2)$ (8)

Rayleigh – Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

□ (V*- T*) = 0 (9)

For arbitrary variations of W satisfying relevant geometrical boundary conditions, the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$\left.\begin{aligned} W = W,_x = 0, \quad x = 0, a \\ W = W,_y = 0, \quad y = 0, a \end{aligned}\right\} \quad (10)$$

Now assuming the non-dimensional variables as

X= X / a, Y= y /a, M= W / a, N=h/a (11)

The kinetic energy T* and strain energy V * are [2]

$$T^* = (1/2)\,\rho p^2 N_0 a^5 \int_0^1 \int_0^1 [(1+□_1X) (1+□_2Y)M^2]dYdX \quad (12)$$

Where, $Q = E_o h_o{}^3a^3/24 (1-v^2)$ (13)

Using equations (12) & (13) in equation (9), one get

$$(V^{**} - □^2 T^{**}) = 0$$

(14)

Where,

$$V^{**} = \int_0^1 \int_0^1 [1- □ (1-X)(1-Y^2)] (1+□_1 X)^3 (1+□_2 Y)^3 \{ (M,_{xx})^2 + (M,_{YY})^2 \qquad (15)$$

$$+2\, vM,_{xx} M,_{YY} + 2 (1-v)(M,_{XY})^2 \} dYdX$$

And

$$T^{**} = \int_0^1 \int_0^1 [(1+□_1 X)(1+□_2 Y) M^2]\, dYdX \qquad (16)$$

Here, □ $□^2 = 12\, \rho (1-v^2) a^2 / E_0 h_0^2$ is a frequency parameter.

Equation (14) consists two unknown constants i.e. A_1 &$_2$ arising due to the substitution of W. These two constants are to be determined as follows

$$\partial(V^{**} - □^2 T^{**}) / □ A_n \quad , \quad n = 1, 2 \qquad (17)$$

On simplifying (17), one gets.

$$bn_1 A_1 + bn_2 A_2 = 0 \quad , \quad n = 1, 2 \qquad (18)$$

Where , bn_1, bn_2 (n=1, 2) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (18) must be zero So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \qquad (19)$$

With the help of equation (19), can obtains a quadratic equation in $□^2$ from which the two values of $□^2$ can found. These two values represent the two modes of vibration of frequency i.e. $□_1$ (Model) & $□_2$for different values of taper constant and thermal gradient for a clamped plate.

Result and Discussion

The frequency equation, in from which two roots can be determined. The frequency parameter & corresponding to the first two modes as vibration of clamped square plate have been computed for various values as temperature gradient (□) and taper constant(□$_□$)

The values of frequency decreases as value of thermal gradient increase from 0.0 to 1.0 for $□_1$=□$_□$=0.0 and $□_1$=□$_□$= 0.6 for both modes of vibrations.

The value of frequency increases as value of takes constant increases from 0.0 to 1.0 for □=0.2 , □$_□$=0.4 and □=0.6, □$_□$= 0.8 respectively.

Conclusion

The present study is an analytical approach to determine the frequencies of non-homogeneous tapered square plate. Actually authors suggested the scientists & mechanical engineers that they have to study the numerical findings of the present paper before finalizing any machine or structure. They can make a use of it with a practical approach, for the welfare of the human beings as well as for the advancement of technology.

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