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Queuing Theory with Customer Impatience

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ABSTRACT

Queuing theory is encountered by so many hurdles. A customer gets impatient by spending a long period of time in a waiting line to get served. The behaviour of a customer plays a vital role in queuing theory. The impatient customers put a negative impact on the queuing system under investigation. If we talk from business point of view the firms lose their potential customers due to the customer impatience, and obviously their business gets affected as a whole. In paper [1] by Rakesh Kumar and Sumeet Kumar Sharma have studied the single server model with discouraged arrivals, reneging and retention of reneged customers. In this paper we shall try to simplify the expressions involving P_n, L_s, L_q, W_s, W_q *.It is also studied that this model quite practical in nature.*

Keywords: Reneging, Probability of customer, single server

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1. Introduction

Queuing theory is considered as the branch of operations research because the results are often used when making business decisions about the resource needed to provide service. Queuing theory is the mathematical study of waiting lines, or queues [2]. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted [2].queuing theory is not too old. It started with research by A.K. Erlang when he created models to describe the Copenhagen telephone exchange [1]. The ideas have since seen applications including telecommunication [3][4], and the design of factories, shops , offices and hospitals[5][6] Queuing theory is concerned with the mathematical modelling and analysis of system that provide service to random demands. Typically the simple queuing models are specified in terms of the arrival process, the service mechanism and the queue discipline [7].The word queue comes from the Latin word *Cauda*, meaning tail. The spelling "*queuing*" over "*queueing*" is typically encountered in the academic research field.

2. Queuing Model Description.

This model is based on the standard Markovian assumptions of inter-arrival and service times. The average arrival rate is and the average service rate is .The capacity of the system is taken as finite, say N. There is a single server. The queue discipline is first–come, first-served (FCFS).Each customer upon arriving in the queue will wait a certain length of time (reneging time) for his service to begin. If it has not begun by then, he will get impatient and may leave the queue without getting service with probability and may remain in the queue for his service with probability $q(=1-p)$. The reneging times follow exponential distribution with parameter ζ .

3. Mathematical Formulation and Solution of the Model.

In this section, the mathematical framework of the queuing model has been presented. The differential-difference equations of the model have been derived by using the general birth-death arguments. These equations have been solved iteratively in steady-state in order to obtain the steady state solution.

Define,

$p_n(t) =$ the probability that there are n customer in the system, that is, $n-1$ in the queue and one in service.

Let $P_n(t)$ be the probability that there are n customers in the system at time t . The difference-differential equations are derived by using the general birth death arguments. These equations are solved iteratively in steady state in order to obtain the steady state solution.

The differential-difference equations of the model are:

$$\frac{dP_0(t)}{dt} = \lambda P_0(t) + \mu P_1(t) \qquad (1)$$

$$\frac{dP_n(t)}{dt} = -\left[\frac{\lambda}{n+1} + \mu + (n-1)\zeta p\right]P_n(t) + (\mu + n\zeta p)P_{n+1}(t) + \frac{\lambda}{n}P_{n-1}(t),$$

$$where N = 1,2,3...N-1 \qquad (2)$$

$$\frac{dP_N(t)}{dt} = \frac{\lambda}{n}P_{N-}(t) - [\mu + (N-1)\zeta p]P_N(t) \qquad (3)$$

In steady state $\lim_{n\to\infty} P_n(t) = P_n$ and therefore $\frac{dP_n}{dt} = 0 \ \ as\, t \to \infty$ and hence the solution of equation (1) to (3) gives the difference equations.

$$0 = \lambda P_0 + \mu P_1 \quad (4)$$

$$0 = -\left[\left(\frac{\lambda}{n+1}\right) + \mu + (n-1)\zeta p\right] P_n + (\mu + n\zeta p) P_{n+1} + \left(\frac{\lambda}{n}\right) P_{n-1}$$

$$where\, n = 1,2..., N-1 \quad (5)$$

$$0 = -[(\mu + (n-1)\zeta p)] P_N + \left(\frac{\lambda}{n}\right) P_{N-1} \quad (6)$$

Solving iteratively equations iteratively (4) to (6) we get

$$p_n = P_0 \left[\frac{1}{n!} \prod_{k=1}^{n} \frac{\lambda}{[\mu + (k-1)\zeta p] P_n}\right];\ 1 \le n \le N \quad (7)$$

Now by using the steady state condition

$$\sum_{n=0}^{N} P_n = 0$$

$$\Rightarrow \sum_{n=1}^{N} P_n = 1 - P_0 \quad , we \quad have$$

$$P_0 = \frac{1}{1 + \sum_{n=1}^{N} \left(\frac{1}{n!} \prod_{k=1}^{n} \frac{\lambda}{[\mu + (k-1)\zeta p]}\right)} \quad (8)$$

Hence the steady state probabilities of the system size are derived explicitly.

4. Measures of effectiveness.

In this section some important measures of effectiveness are derived. These can be used to study the performance of the queuing system under consideration.

The expected System size L_s

$$L_S = \sum_{n=1}^{N} n \left(\frac{1}{n!} \prod_{k=1}^{n} \frac{\lambda}{[\mu + (k-1)\zeta p]}\right) P_0$$

The expected queue length(L_q)

$$L_q = L_s - \frac{\lambda}{\mu} = \left[\sum_{n=1}^{N} n \left(\frac{1}{n!} \prod_{k=1}^{n} \frac{\lambda}{[\mu + (k-1)\zeta p]} \right) P_0 - \frac{\lambda}{\mu} \right]$$

The expected waiting time in the system W_S

$$W_s = \left[\frac{1}{\lambda} \sum_{n=1}^{N} n \left(\frac{1}{n!} \prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\zeta p} \right) \right] P_0$$

The expected waiting time in the queue W_q

$$W_q = \left[\frac{1}{\lambda} \sum_{n=1}^{N} n \left(\frac{1}{n!} \prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\zeta p} \right) P_0 - \frac{1}{\mu} \right]$$

The expected number of customers served E

$$E = \sum_{n=1}^{N} n\mu P_n = \sum_{n=1}^{N} n\mu \left(\frac{1}{n!} \prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\zeta p} \right) P_0$$

Rate of abandonment R_{aband}

$$R_{aband} = \lambda \sum_{n=0}^{N} P_n - E$$
$$= \sum_{n=1}^{N} n\mu \left(\frac{1}{n!} \prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\zeta p} \right) P_0$$

Expected number of waiting customers, who actually wait

$$E_w = \frac{\sum_{n=2}^{N} (n-1) P_n}{\sum_{n=2}^{N} P_n}$$
$$= \frac{\sum_{n=2}^{N} (n-1) \left[\frac{1}{n!} \prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\zeta p} \right] P_0}{\sum_{n=2}^{N} \left[\frac{1}{n!} \prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\zeta p} \right] P_0}$$

Probability distribution of busy period

Prob.(busy period) =Prob. $(n \geq 1)$

Prob.(busy period) = $\sum_{n=1}^{N}\left[\frac{1}{n!}\prod_{k=1}^{n}\frac{\lambda}{\mu+(k-1)\zeta p}\right]P_0$

Special cases

When there is no retention of reneged customer (when q=0).

The queuing system is reduced to a system with discouraged arrivals and reneging with $P_n=\left[\frac{1}{n!}\prod_{k=1}^{n}\frac{\lambda}{\mu+(k-1)\zeta p}\right]P_0; i\le n\le N$

Using the steady state condition

$$P_0+P_1+P_2+...+P_n=1$$
$$\Rightarrow\sum_{n=0}^{N}P_n=1, \textit{ we get}$$

$$P_0=\frac{1}{1+\sum_{n=1}^{N}\left(\frac{1}{n!}\prod_{k=1}^{n}\frac{\lambda}{[\mu+(k-1)\zeta p]}\right)}$$

When there is no discouragement.

We study two sub-cases:

The model reduces to an M/M/1/N queuing model with retention of reneged customers with

$$P_n=\prod_{k=1}^{n}\frac{\lambda}{[\mu+(k-1)\zeta p]}P_0; 1\le n\le N-1$$

Also for n=N we have

$$P_N=\prod_{k=1}^{N}\frac{\lambda}{[\mu+(k-1)\zeta p]}P_0 .$$

Using the steady state condition

$$P_0+P_1+P_2+...+P_n=1$$
$$\Rightarrow\sum_{n=0}^{N}P_n=1, \textit{ we get}$$

$$P_0 = \frac{1}{1+\sum_{n=1}^{N}\left(\prod_{k=1}^{n}\frac{\lambda}{\mu+(k-1)\zeta p}\right)}$$

2. When there is no reneging.

In this case the probability of reneging (p) is zero implies that $\zeta = 0$. As there is no reneging so there is not any kind of retention of customers. All the customers who enter into the system leave after getting service. Therefore from equations (7) and (8) it is quite clear that

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 ; 1 \le n \le N$$

And using the normalization condition, $\sum_{n=0}^{N} P_n = 1$, *we get*

$$P_0 = \frac{1}{1+\sum_{n=0}^{N}\left(\frac{\lambda}{\mu}\right)^n}$$

Obviously the model reduces to a simple single server queuing model /M/1/N with Poison distribution.

Conclusion.

In this paper we come to know that the model is confined to finite queuing capacity. This paper studies the effect of reneged behaviour of the model on the queue. Further the model can be solved in transient state to get time dependent results. This paper is also very useful in solving the queuing problems in terms of reneged customers. From the customer point of view the model is efficient and practical.

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